

FIELD THEORETICAL METHODS IN ELEMENTARY PARTICLE PHYSICS

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TO MY PARENTS

The thesis is based on the following articles:

- I. Stationary Phase Methods in Minkowski Space with Application to Asymptotic Estimates in Perturbation Theory.
Lund Preprint LU TP 78-4.
- II. Application of Low Energy Theorems to the New Particles.
Physics Letters 60B (1976) 465.
(Together with H.R. Rubinstein)
- III. Compton Scattering, Low Energy Theorems and Sum Rules.
Lund Preprint LU TP 78-2.
- IV. Magnetic Charge In a Pure Yang-Mills Theory.
Lund Preprint LU TP 78-3.
- V. Possibly Non-Resonant Structure in Argand Plots in K^+p Scattering.
Lund Preprint Aug. 1972.

INTRODUCTION

The creation of the theory of relativity in the beginning of this century and a little later the introduction of quantum mechanics turned out to be very successful developments for the understanding of basic properties of matter and its fundamental interactions. The classical theory of electromagnetism formulated more than a century ago by Maxwell was very successful in describing macroscopic phenomena but failed when applied to the atomic world. It however had the right properties for being a relativistic theory, and in the late 1940's it was reformulated into an acceptable quantum theory, in modern terms a quantum field theory. This so-called quantum electrodynamics (QED) agrees extremely well with experiment. The methods of calculation are in most cases perturbative, with the expansion parameter being the so-called fine structure constant α , whose value is approximately $1/137$.

Although being in good agreement with experiment, the perturbative solution of QED suffers from a fundamental difficulty: the series diverges. This was actually recognized long ago by Dyson, but it is not until recently that the problem has been discussed in a more detailed manner, mainly due to Lipatov. It turns out that most perturbative solutions in quantum mechanics give rise to divergent series. This, for instance, includes the problem of the anharmonic oscillator potential. The important property of perturbation expansions is however that the series is summable. Technically it means that there are ways to define the formally divergent series to get a finite answer. This is clearly a very important property and certainly adds more confidence to using quantum field theory. Paper I discusses some aspects of how to deduce the summability of a certain field theoretical model.

A very basic ingredient in QED is the property of being gauge invariant. It means that the Lagrangian (and Hamiltonian) is invariant under a change of the phase of the fermion field by a coordinate dependent function, provided that the photon vector field at the same time undergoes a transformation, essentially a translation equal to the gradient of the change of the phase function. Gauge invariance leads to electromagnetic current conservation and this in turn implies many restrictions on scattering amplitudes, form factors etc. Some of these constraints give rise to certain sum rules, a few of which are discussed in papers II and III, where they are used for discussing properties of the newly discovered particles in the so-called ψ -family.

In the case of QED the class of gauge transformations form an abelian group. Not long after the formulation of QED, Yang and Mills introduced an extension to the case of non-abelian gauge transformations. In the simplest version, with the group $SU(2)$ as gauge group, the fermion fields are in a doublet representation. Analogous to the case of QED, the interaction is mediated by massless vector fields, and for the case of $SU(2)$ there are three of them. The properties of these vector particles are however of fundamentally different nature than in QED. Namely, in a non-abelian gauge theory they themselves carry charge. This also means that there are nonlinear selfinteractions between them without any fermions involved, a fact which makes the theory considerably more complicated than QED.

During the last ten years non-abelian gauge theories have proved to be very useful for describing another important interaction in nature, the weak interaction. The earlier theory of weak interactions suffered from a severe problem by not being unitary. This is now completely cured and at the same time there has been a kind of unification of the weak and electromagnetic interactions. In

one of the first models, the Weinberg-Salam model with gauge group $SU(2) \times U(1)$, the two interactions are combined into one theory. It not only describes the old results in a fully consistent way, but also predicts a new kind of current, the so-called neutral current, which was later found experimentally.

When it comes to the third kind of fundamental force in nature, the strong interaction, it is fair to say that quantum field theory has so far not been able to provide much quantitative information. There are, however, several ideas and general properties of the strong interaction that originate in field theory, and once more non-abelian gauge theories have provided a framework. There is good experimental evidence that one can describe all strongly interacting particles, the hadrons, in terms of a few basic constituents, the so-called quarks. Each of the quarks (at present there is good evidence for at least four and maybe five different ones) is believed to come in three different varieties corresponding to an internal symmetry group of $SU(3)$, popularly called the color group of the strong interaction. This group is taken as basis for a non-abelian gauge theory called quantum chromodynamics (QCD). In QCD the quarks are fermions that interact via eight "colored" gluons, which are the massless vector particles of the gauge theory. The hadrons that we observe in nature are supposed to be built up of bound states of the quarks. It turns out that one can account for all observed hadrons by assuming that they are singlet states under the $SU(3)$ gauge group. There is a general belief, although not yet proven, that all the observable effects of the theory correspond to various gauge invariant properties of it. Thus physical states, scattering amplitudes etc. should be invariant under local gauge transformations in color space. This line of argument has recently led to the discovery of different topological properties of gauge theories in general. It turns out that one can classify some of the properties in a gauge invariant way, giving rise to among other things

topologically conserved charges. Such an attempt is discussed in paper IV for the case of a pure non-abelian gauge theory of $SU(2)$ without fermions.

QCD has had some experimental success. It very well describes the deep inelastic lepton-hadron experiments, where electrons, muons and neutrinos are colliding with nucleons at high energy and large momentum transfer. The so-called scaling property of the data is predicted by the theory, which also gives the deviations from scaling that are observed experimentally. This result is a direct consequence of the non-abelian nature of QCD and follows from the possibility to make a perturbation expansion in an effective coupling constant which decreases with momentum transfer (asymptotic freedom).

As already mentioned, quantum field theory is a synthesis of quantum mechanics and relativity theory. This among other things leads to the requirement that the Lagrangian has to be invariant under Lorentz transformations. Another very general property is the invariance of any local quantum field theory under a combined transformation of parity, charge conjugation and time-reversal operations. Some interactions are invariant under each of these separately, like electrodynamics and the strong interaction. In general, various symmetries of an interaction can relatively easily be built into the Lagrangian, and this sometimes leads to general predictions of scattering amplitudes that would be difficult to deduce without the aid of field theory. In papers II, III and V, for example, various effective couplings between particles are conveniently deduced by writing down field-theoretical Lagrangians with the required symmetry properties.

For many years dispersion relations have been a very important tool for both more axiomatic and phenomenological studies in

hadron physics. An example of an application of them is presented in paper V. Also the sum rules discussed in papers II and III have their origin in dispersion relations. The fact that one can write dispersion relations for scattering amplitudes and other physical quantities has its roots in quantum field theory. In fact, it is only within the framework of this theory that it is possible to give rigorous proofs for dispersion relations. Crucial for this are the causality properties that follow from the relativistic invariance. (One should emphasize that all this is only true in local, renormalizable theories, which are the only ones we are considering here.) Dispersion relations are manifestations of the general singularity structure of scattering amplitudes such as the presence of various poles and cuts, which are most easily seen within the context of a field theory. It is true that many of these properties can be deduced from so-called pure S-matrix theory, but there is no doubt that field theory has played an important role also for the development of this approach.

For many years during the 1960's and the beginning of the 70's field theory was by many people considered an impractical tool for the understanding of elementary particle physics in general and for hadron physics in particular. With gauge theories the state of affairs seems to have changed. The old objection to using perturbation theory in hadron physics because of the large coupling constant is not as valid any more. The asymptotically free property of non-abelian gauge theories in certain high energy limits makes the effective coupling constant small so that perturbation theory can be used. On the basis of old experimental properties of the weak interaction it was found that a simple way to accommodate them within a quantum field theory is to introduce a new quantum number in terms of an additional quark. The recent discovery of particles indeed carrying such a quantum number (charm) lends direct support to this scheme. It is in fact the first time a new quantum number has been predicted and later also found.

DISCUSSION OF PAPERS

Paper I.

Use of the path integral in quantum mechanics can often give results that would be difficult or laborious to obtain in any other way. It has therefore become an important tool in spite of the fact that it is not always mathematically rigorous. A relativistic quantum field theory is formulated in Minkowski space with an action $S = \int d^4x L$, where L is the Lagrangian density expressed in terms of the field variables. The weight factor in the path integral is given by $\exp(iS)$. Although such an integral eventually has to be used to calculate physical quantities, it is not always clear how to mathematically perform the calculations since the weight factor is not exponentially damped for large S . In many cases one is mainly interested in some properties of Greens- functions, and for this purpose it has been proposed to reformulate the path integral in Euclidean space-time by making a so-called Wick rotation. This means that time x_0 is replaced by $-ix_4$. The expression iS is then converted into $-A$, where A is a positive definite functional of the field variables, which in turn are functions of the space coordinates and x_4 . Recently Lipatov suggested to estimate the behavior of Green's functions for large orders in perturbation theory by projecting out the k th order from the Euclidean functional integral and applying a saddle point approximation to evaluate it (for k large). When this technique is applied to for example a real scalar field ϕ with selfinteraction $g\phi^4$ one finds that the coefficient of g^k of some Green's function behaves as $(-1)^k k! a^k b^k c$ for large k . The $(-1)^k$ implies that the perturbation series is summable in spite of the fact that it is formally divergent. This is actually a general property of observables expressed as a perturbation series in a renormalizable theory. A convenient way to sum it is by using a technique due to Borel.

Since the path integral originally is formulated in Minkowski space it would be gratifying if the same results could be obtained directly there. In paper I we show that this is possible. In that case one has to use a stationary phase approximation (because of the i in $\exp(iS)$) for evaluating the path integral. In order to get solutions to the stationary phase equations with finite action S , it is necessary to consider complex solutions in spite of the fact that the original path integral is over real fields. The mathematical justification for this is admittedly not as rigorous as for the saddle point method in the Euclidean case. The result for the asymptotic behavior is, however, analogous to that case, and therefore one can also in this way obtain the summability of the perturbation series. The complex stationary phase solution which gives this result has an interesting property in that it corresponds to a field configuration with the same zero energy as the vacuum but with a finite nonzero action.

Papers II and III.

The predicted new quantum number charm first revealed itself experimentally with the discovery of the very narrow spin one resonances ψ and ψ' at 3.1 and 3.7 GeV respectively. The production mechanism is essentially electromagnetic via a virtual time-like photon as is most easily seen in e^+e^- -annihilation. Soon after this discovery three other states in between were found arising from electromagnetic decays of ψ' and in turn partially decaying electromagnetically into ψ . Also a state below the mass of the ψ has been seen, obtained from it via a one photon decay. In paper II the different electromagnetic decay widths are discussed with the attempt to relate them to each other in a model independent way. In low energy Compton scattering on ψ one expects to find mainly low excited states of the same type as ψ , namely states composed of a pair of charm-anticharm quarks. A reasonable assumption is therefore that the Compton cross section should be

saturated by coupling to a few intermediate states. Since ψ has negative charge conjugation, these states would have positive charge conjugation. The further assumption is that the four states mentioned above are exactly those needed for saturation.

From dispersion relations and unitarity one can obtain a relation connecting the difference between the photon cross section for the photon polarization parallel respectively antiparallel to the target spin, and the value of the Compton scattering amplitude at small photon energy. The latter quantity can be calculated by using gauge invariance and is related to the magnetic moment of the particle. For neutral particles that are assumed to be eigenstates of charge conjugation this vanishes. One then obtains a sum rule saying that an energy integral over the difference between the parallel and antiparallel photon-target cross section (divided by the photon energy) should be zero. Saturating this integral with the positive charge conjugation states as intermediate states gives a relation between the different couplings of them to the ψ -photon system. This immediately implies a relation between their various electromagnetic decay widths. Thus one can predict that the ratio between the electromagnetic width of an assumed spin 2, positive parity state (with mass in the range 3.3 to 3.5 GeV) to the width of ψ going into a spin zero, negative parity state at 2.8 GeV should be in the range

$$0.2 \leq \Gamma_{2 \rightarrow \psi \gamma} / \Gamma_{\psi \rightarrow 0 \gamma} \leq 1.5$$

depending on the mass of the spin 2 state.

One can in a similar way discuss Compton scattering on the spin 2 state assuming saturation with a few intermediate states with negative charge conjugation like ψ and ψ' . It turns out that one in addition needs at least a spin 3 state. If the mass of this state is around 4 GeV, one gets a lower bound for the width of $3 \rightarrow 2 + \gamma$ in terms of the widths $\psi' \rightarrow 2 + \gamma$ and $2 \rightarrow \psi + \gamma$. As an example, if the mass of the 2 state is 3.5 GeV one obtains

$\Gamma_{3 \rightarrow 2 \gamma} \gtrsim 13 \Gamma_{\psi' \rightarrow 2 \gamma} + \Gamma_{2 \rightarrow \psi \gamma}$. This high spin state should then be possible to see experimentally.

Paper IV.

A few years ago 't Hooft and Polyakov showed that a spontaneously broken $SO(3)$ Yang-Mills theory with only a triplet of scalar fields interacting via the gauge vector fields admits a static finite energy solution with a characteristic topologically conserved charge. This charge has the important property of being invariant under arbitrary non-abelian gauge transformations. The field configurations giving rise to this can be viewed as a kind of extended particle with the property of a magnetic charge (monopole). During the last few years there has been considerable interest in basic properties of pure Yang-Mills theories where only the non-abelian gauge fields are present. Several investigations have been concerned with solving the classical field equations in Minkowski space. Some years ago Wu and Yang presented a static solution to them. However, this one actually corresponds to a source at the origin, and in addition does not have finite energy. In fact, in order to get finite energy one has to consider time dependent solutions. So far essentially only one such solution (corresponding to real fields) has been obtained for the case of the gauge group $SU(2)$. An obvious question about solutions in general concerns the physical meaning of them. A first step in the direction of answering this is to understand their possible gauge invariant properties.

Such an attempt is discussed in paper IV. By projecting the Lorentz indices of the non-abelian field tensor in a special way it is possible to define a triplet of scalar fields that transform as the vector representation of the gauge group. The scalar fields can then be used in the same way as was done by 't Hooft to define a gauge invariant tensor which immediately gives rise to a conserved topological charge. The same procedure works for both the Wu-Yang solution and the finite energy one. In both cases one obtains a charge of $1/g$, g being the gauge coupling constant. This is also seen directly from the gauge invariant tensor, which has the geometrical property of a magnetic charge of $1/g$ at the origin.

Paper V.

According to the standard quark model the observed baryons are composed of three quarks. This immediately leads to the conclusion that for example a resonance state with the quantum numbers of the system K^+p cannot be described within the ordinary quark model. For this reason such a resonance is called exotic. A lot of experimental effort has been made searching for exotic states, both baryonic and mesonic ones. A possible candidate is a direct resonance in K^+p scattering for the partial wave with $\ell=1$, $J=3/2$. The experimental elastic phase shifts for this reaction show an Argand plot with a loop-like behavior.

It has been shown that the main inelastic channel for low energies is the $K\Delta$ -channel. This is a good approximation up to 1.5 GeV (lab. energy), but above that the K^*N channel becomes important. In paper V we have restricted the inelastic channels to only the $K\Delta$ -system since the experimental analysis of the reaction shows that a possible resonance is not far above 1.5 GeV. To take into account unitarity for this two-channel problem, the technique of N/D equations was used. The two by two scattering matrix is written as $N \cdot D^{-1}$, where N and D are 2×2 matrices containing the left respectively right-hand singularities in the variable s . Assuming that the left-hand singularities can be approximated by a few poles and by using ordinary dispersion relations, one can solve the unitarity equations to obtain the residues of these poles. From this it is straightforward to calculate the matrix D and the scattering matrix itself.

If there is a resonance it should show up in the P_3 -wave by giving a small value for the real part of the determinant of D . This is not the case for our qualitative solution. D is normalized to the unity matrix at some energy, and $\text{Re}(\det D)$ stays close to one in the whole range of energies investigated. Another requirement in order to have a resonance is that the elastic amplitude should

change most rapidly around the tentative resonance energy. This is not found to be the case. On the contrary the changes become slower the closer one approaches the presumed resonance position. An analogous calculation using instead a simple p -exchange for the inelastic reaction $KN \rightarrow K\Delta$ leads to similar results. The conclusion is therefore that the qualitative fit presented to the data does not seem to correspond to a resonance.

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